

Brans-Dicke Theory and Thermodynamical Laws on Apparent and Event Horizons

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In this work, we have described the Brans-Dicke theory of gravity and given a particular solution by choosing a power law form of scalar field ϕ and constant ω . If we assume first law and entropy formula on apparent horizon then we recover Friedmann equations. Next, assuming first law of thermodynamics, the validity conditions of GSL on event horizon are presented. Also without use first law, if we impose the entropy relation on the horizon, then we also obtain the condition of validity of GSL on event horizon. The validity of GSL completely depends on the model of BD scalar field solutions. We have justified that on the apparent horizon the two process are equivalent, but on the event horizon they are not equivalent. If first law is valid on the event horizon then GSL may be satisfied in BD solution, but if first law is not satisfied then GSL is not satisfied in BD solution. So first law always favours GSL on event horizon. In our effective approach, the first law and GSL is always satisfied in apparent horizon, which do not depend on BD theory of gravity.

I. INTRODUCTION

The luminosity-redshift relation observed for type-Ia supernovae [1, 2] strongly suggests that, in the present phase, the universe is undergoing an accelerated expansion. This is supported also by recent measurements of CMBR and the power spectrum of mass perturbations. There are several proposals regarding this, it cosmological constant and quintessence like dark energy [3 - 5] being some of the competent candidates. Basically quintessence is a dynamical slowly evolving spatially inhomogeneous component of energy density with negative pressure. The vector and tensor fields describing the fundamental forces, there may exist scalar field. The energy density associated with a scalar field ϕ slowly moving down its potential V can represent a simple example of quintessence. Another simplest alternative which includes the scalar field in addition to the tensor field in general relativity is Brans-Dicke (BD) theory. Brans-Dicke theory has been proved to be very effective regarding the recent study of cosmic acceleration [6]. BD theory is explained by a scalar function ϕ and a constant coupling constant ω , often known as the BD parameter. This can be obtained from general theory of relativity (GR) by letting $\omega \rightarrow \infty$ and $\phi = \text{constant}$ [7]. This theory has very effectively solved the problems of inflation and the early and the late time behaviour of the Universe. BD scalar-tensor theory can potentially solve the quintessence problem [6]. The generalized BD theory [8] is an extension of the original BD theory with coupling function ω is a function of the scalar field ϕ . Bertolami and Martins [9] have used this theory to present an accelerated Universe for spatially flat model. All these theories conclude that ω should have a low negative value in order to solve the cosmic acceleration problem. This contradicts the solar system experimental bound $\omega \geq 500$. However they have obtained the solution for accelerated expansion with a potential ϕ^2 and large $|\omega|$, although they have not considered the positive energy conditions for the matter

and scalar field. In context of accelerating expansion of the universe, there are several works on BD theory [10, 11] in both theoretical and observational point of views.

Motivated by the black hole physics, it was realized that there is a profound connection between gravity and thermodynamics. In Einstein gravity, the evidence of this connection was first discovered in [12] by deriving the Einstein equation from the proportionality of entropy and horizon area together with the first law of thermodynamics in the Rindler spacetime. The horizon area (geometric quantity) of black hole is associated with its entropy (thermodynamical quantity), the surface gravity (geometric quantity) is related with its temperature (thermodynamical quantity) in black hole thermodynamics [13]. In 1995, Jacobson [12] was indeed able to derive Einstein equations by applying the first law of thermodynamics $\delta Q = TdS$ together with proportionality of entropy to the horizon area of the black hole. He assumed that this relation holds for all Rindler causal horizons through each space time point with δQ and T interpreted as the energy flux and temperature seen by an accelerated observer just inside the horizon. Then Padmanabhan [14] was able to formulate the first law of thermodynamics on the horizon, starting from Einstein equations for a general static spherically symmetric space time.

Frolov and Kofman in [15] employed the approach proposed by Jacobson [12] to a quasi-de Sitter geometry of inflationary universe, where they calculated the energy flux of a background slow-roll scalar field (inflaton) through the quasi-de Sitter apparent horizon and used the first law of thermodynamics $-dE = TdS$, where dE is the amount of the energy flow through the apparent horizon. Although the topology of the local Rindler horizon in Ref. [12] is quite different from that of the quasi-de Sitter apparent horizon considered in Ref. [15], it was found that this thermodynamic relation reproduces one of the Friedmann equations with the slow-roll scalar field. It is assumed in their derivation that $T = \frac{H}{2\pi}$ and $S = \frac{\pi}{GH^2}$ where H is a slowly varying Hubble parameter. Also the identity between Einstein equations and thermodynamical laws has been applied in the cosmological context considering universe as a

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thermodynamical system bounded by the apparent horizon (RA). Using the Hawking temperature $T_A = \frac{1}{2\pi R_A}$ and Bekenstein entropy $S_A = \frac{\pi R_A^2}{G}$ (R_A is the radius of apparent horizon) at the apparent horizon, the first law of thermodynamics (on the apparent horizon) is shown to be equivalent to Friedmann equations [16] and the generalized second law of thermodynamics (GSLT) is obeyed at the horizon. The thermodynamics in de Sitter spacetime was first investigated by Gibbons and Hawking in [17]. In a spatially flat de Sitter spacetime, the event horizon and the apparent horizon of the Universe coincide and there is only one cosmological horizon. In the usual standard big bang model a cosmological event horizon does not exist. But for the accelerating universe dominated by dark energy, the cosmological event horizon separates from that of the apparent horizon. When the apparent horizon and the event horizon of the Universe are different, it was found that the first law and generalized second law (GSL) of thermodynamics hold on the apparent horizon, while they break down if one considers the event horizon [18]. On the basis of the well known correspondence between the Friedmann equation and the first law of thermodynamics of the apparent horizon, Gong et al [19] argued that the apparent horizon is the physical horizon in dealing with thermodynamics problems.

Recently, it is of great interest to study the generalized second law (GSL) of thermodynamics in the generalized gravity theories. There have been a lot of interest on investigating the GSL in gravity [20,21], but all of them concentrate on the Einstein gravity. The modified theory of gravity was argued to be a possible candidate to explain the accelerated expansion of our universe, thus it is interesting to examine the GSL in the extended gravity theories. Even for the Einstein gravity, it was found that GSL breaks down in phantom-dominated universe in the presence of Schwarzschild black hole [22]. Entropy of the horizon from the first law of thermodynamics constructed in [20]. The total entropy evolution with time including the horizon entropy, the non-equilibrium entropy production, and the entropy of all matter, field and energy components have been discussed by Wu et al [23]. They have shown a universal condition to protect the GSL in generalized gravity theories and its validity in the Einstein gravity (even in the presence of Schwarzschild black hole) and higher order gravity. Also there are several works on thermodynamics due to Gauss-Bonnet gravity [24], Horava-Lifshitz gravity [25], Lovelock gravity [16, 23, 26], braneworld gravity [23, 26, 27], $f(R)$ gravity [23, 28, 29] and scalar-tensor gravity [23, 28].

In this work, we briefly describe the Brans-Dicke theory of gravity and give a particular solution by choosing a power law form of scalar field ϕ and constant ω in section II. Next, assuming first law of thermodynamics, the validity conditions of GSL on event horizon are presented in section III. Also without use first law if we impose the entropy relation on the horizon, then we also obtain the condition of validity of GSL on event horizon in section IV. There are two ways to get validity conditions of GSL on apparent and event horizons: (i)

use first law and find entropy relation on the horizons (ii) use only horizon entropy on the horizons. Finally some concluding remarks have been presented in last section.

II. BRANS-DICKE THEORY

The self-interacting Brans-Dicke (BD) theory is described by the Jordan-Brans-Dicke (JBD) action [30] : (choosing $c = 1$)

$$S = \int \frac{d^4x \sqrt{-g}}{16\pi} \left[\phi R - \frac{\omega(\phi)}{\phi} \phi_{,\alpha} \phi_{,\alpha} - V(\phi) + 16\pi \mathcal{L}_m \right] \quad (1)$$

where $V(\phi)$ is the self-interacting potential for the BD scalar field ϕ and $\omega(\phi)$ is modified version of the BD coupling parameter which is a function of ϕ . In this theory $\frac{1}{\phi}$ plays the role of the gravitational constant G . This action also matches with the low energy string theory action [31] for $\omega = -1$. The matter content of the Universe is composed of matter fluid, so the energy-momentum tensor is given by

$$T_{\mu\nu}^m = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \quad (2)$$

where u^μ is the four velocity vector of the matter fluid satisfying $u_\mu u^\mu = -1$ and ρ , p are respectively energy density and isotropic pressure.

From the Lagrangian density (1) we obtain the field equations [11]

$$G_{\mu\nu} = \frac{8\pi}{\phi} T_{\mu\nu}^m + \frac{\omega(\phi)}{\phi^2} \left[\phi_{,\mu} \phi_{,\nu} - \frac{1}{2} g_{\mu\nu} \phi_{,\alpha} \phi_{,\alpha} \right] + \frac{1}{\phi} [\phi_{,\mu;\nu} - g_{\mu\nu} \square \phi] - \frac{V(\phi)}{2\phi} g_{\mu\nu} \quad (3)$$

and

$$\square \phi = \frac{8\pi T}{3 + 2\omega(\phi)} - \frac{1}{3 + 2\omega(\phi)} \left[2V(\phi) - \phi \frac{dV(\phi)}{d\phi} \right] - \frac{\frac{d\omega(\phi)}{d\phi}}{3 + 2\omega(\phi)} \phi_{,\mu} \phi^{,\mu} \quad (4)$$

where $T = T_{\mu\nu}^m g^{\mu\nu}$. Equation (3) can also be written as

$$G_{\mu\nu} = 8\pi \tilde{T}_{\mu\nu} = \frac{8\pi}{\phi} (T_{\mu\nu}^m + \frac{1}{8\pi} T_{\mu\nu}^\phi) \quad (5)$$

where $\tilde{T}_{\mu\nu}$ can be treated as effective energy momentum tensor. The line element for Friedman-Robertson-Walker space-time is given by

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (6)$$

where, $a(t)$ is the scale factor and k ($= 0, -1, +1$) is the curvature index describe the flat, open and closed model of the universe.

The Einstein field equations and the wave equation for the BD scalar field ϕ are in the following [11]

$$H^2 + \frac{k}{a^2} = \frac{8\pi\rho}{3\phi} - H\frac{\dot{\phi}}{\phi} + \frac{\omega(\phi)}{6}\frac{\dot{\phi}^2}{\phi^2} + \frac{V(\phi)}{6\phi} \quad (7)$$

$$2\dot{H} + 3H^2 + \frac{k}{a^2} = -\frac{8\pi p}{\phi} - \frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} - 2H\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} + \frac{V(\phi)}{2\phi} \quad (8)$$

and

$$\ddot{\phi} + 3H\dot{\phi} = \frac{8\pi(\rho - 3p)}{3 + 2\omega(\phi)} + \frac{1}{3 + 2\omega(\phi)} \left[2V(\phi) - \phi \frac{dV(\phi)}{d\phi} \right] - \frac{\frac{d\omega(\phi)}{d\phi}}{3 + 2\omega(\phi)} \quad (9)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Now let us assume the matter is conserved in BD theory. So the matter conservation equation is given by

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (10)$$

Equations (7) and (8) can be written as

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho_{eff} \quad (11)$$

and

$$2\dot{H} + 3H^2 + \frac{k}{a^2} = -8\pi p_{eff} \quad (12)$$

where ρ_{eff} and p_{eff} are effective fluid density and pressure for combination of matter and the contribution of BD field respectively defined by

$$\rho_{eff} = \frac{\rho}{3\phi} + \frac{3}{8\pi} \left(-H\frac{\dot{\phi}}{\phi} + \frac{\omega(\phi)}{6}\frac{\dot{\phi}^2}{\phi^2} + \frac{V(\phi)}{6\phi} \right) \quad (13)$$

$$p_{eff} = \frac{p}{\phi} + \frac{1}{8\pi} \left(\frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} + 2H\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} - \frac{V(\phi)}{2\phi} \right) \quad (14)$$

From (11) and (12), we see that the field equations are same as the usual Friedmann equations in Einstein gravity. If we use the wave equation (9) and the matter conservation equation (10), we obviously obtain the conservation equation of effective fluid as

$$\dot{\rho}_{eff} + 3H(\rho_{eff} + p_{eff}) = 0 \quad (15)$$

Here we consider the Universe to be filled with barotropic fluid with EOS

$$p = w\rho \quad (-1 < w < 1) \quad (16)$$

The conservation equation (10) yields the solution for ρ as,

$$\rho = \rho_0 a^{-3(1+w)} \quad (17)$$

where $\rho_0 (> 0)$ is an integration constant.

Let us choose $\omega(\phi) = \omega = \text{constant}$. In this case we consider only one power law form of ϕ as

$$\phi = \phi_0 a^\alpha \quad (18)$$

Using equation (16) and (17) and solving equations (7) and (8) we get

$$H^2 = kAa^{-2} + Ba^{-\frac{2\alpha((1+\omega)\alpha-1)}{2+\alpha}} + Ca^{-\alpha-3(1+w)} \quad (19)$$

$$\dot{H} = kA_1a^{-2} + B_1a^{-\frac{2\alpha((1+\omega)\alpha-1)}{2+\alpha}} + C_1a^{-\alpha-3(1+w)} \quad (20)$$

and

$$V = kA_2a^{\alpha-2} + B_2a^{\alpha-\frac{2\alpha((1+\omega)\alpha-1)}{2+\alpha}} + C_2a^{-3(1+w)} \quad (21)$$

where $A = \frac{2}{\alpha((1+\omega)\alpha-1)-(2+\alpha)}$, $C = -\frac{16\pi(1+w)\rho_0}{\phi_0[(2+\alpha)(-\alpha-3(1+w))+2\alpha((1+\omega)\alpha-1)]}$, $A_1 = \frac{2+\alpha(1-\alpha(1+\omega))A}{2+\alpha}$, $B_1 = \frac{\alpha(1-\alpha(1+\omega))B}{2+\alpha}$, $C_1 = \frac{\alpha(1-\alpha(1+\omega))\phi_0 C - 8\pi(1+w)\rho_0}{(2+\alpha)\phi_0}$, $A_2 = 6\phi_0 + AB_2/B$, $B_2 = 6\pi_0(1 + \alpha - \omega\alpha^2/6)B$, $C_2 = B_2C/B - 16\pi\rho_0$ and B is an arbitrary constant.

III. STUDY OF THERMODYNAMICS ON APPARENT AND EVENT HORIZONS

In this section, we assume that the first law is valid [32] on apparent/event horizons and after that we examine the validity of GSL on apparent/event horizons.

From the first law of thermodynamics, we get the relation [16, 33]

$$\begin{aligned} T_X dS_X &= -dE_X = 4\pi R_X^3 H \tilde{T}_{\mu\nu} k^\mu k^\nu dt \\ &= 4\pi R_X^3 H (\rho_{eff} + p_{eff}) dt \end{aligned} \quad (22)$$

where suffix X denotes the apparent horizon ($X = A$) and event horizon ($X = E$). Also T_X and R_X are the temperature and radius of apparent/event horizons. The radii of apparent and event horizons are defined by

$$R_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}} \quad (23)$$

and

$$R_E = a \int_a^\infty \frac{da}{a^2 H} \quad (24)$$

which immediately give,

$$\dot{R}_A = -HR_A^3 \left(\dot{H} - \frac{k}{a^2} \right) \quad (25)$$

and

$$\dot{R}_E = HR_E - 1 \quad (26)$$

Now from equation (22) we get the rate of change of entropy on the apparent/event horizon as

$$\dot{S}_X = \frac{4\pi R_X^3 H}{T_X} (\rho_{eff} + p_{eff}) \quad (27)$$

To study GSL of thermodynamics through the universe we deduce the expression for normal entropy using the Gibb's equation of thermodynamics [18]

$$T_X dS_{IX} = p_{eff} dV + d(E_{IX}) \quad (28)$$

where, S_{IX} is the internal entropy within the apparent/event horizon. Here the expression for internal energy can be written as $E_{IX} = \rho_{eff} V_X$, where the volume of the sphere is $V_X = \frac{4}{3}\pi R_X^3$. So from equation (28), we obtain the rate of change of internal energy as

$$\dot{S}_{IX} = \frac{4\pi R_X^2}{T_X} (\rho_{eff} + p_{eff}) (\dot{R}_X - HR_X) \quad (29)$$

Adding (27) and (29), the rate of change of total entropy is obtained as

$$\dot{S}_{tX} = \dot{S}_X + \dot{S}_{IX} = \frac{4\pi R_X^2 \dot{R}_X}{T_X} (\rho_{eff} + p_{eff}) \quad (30)$$

Using (11), (12), (25) and (30), we see that on the apparent horizon $\dot{S}_{tA} \geq 0$ always. So on the apparent horizon the GSL is satisfied. So validity of GSL on apparent horizon do not depend on BD theory of gravity. But on the event horizon the GSL will be satisfied if any one of the following conditions hold: (i) $\dot{R}_E \geq 0$ and $\rho_{eff} + p_{eff} \geq 0$ or (ii) $\dot{R}_E \leq 0$ and $\rho_{eff} + p_{eff} \leq 0$. Here, the inequalities completely depend on BD scalar field solutions. So validity of GSL on event horizon completely depends on BD theory of gravity. From (13) and (14), we see that $(\rho_{eff} + p_{eff})$ is independent of V . So the validity of GSL depends only on matter and scalar field, but not its potential. Now using BD solutions (17)-(20), we draw the rate of change of total entropy on the event horizon i.e., \dot{S}_{tE} against z in figure 1 for open, closed and flat models. From the figure it is to be seen that when z decreases, \dot{S}_{tE} becomes positive. So we conclude that BD field supports to GSL of thermodynamics.

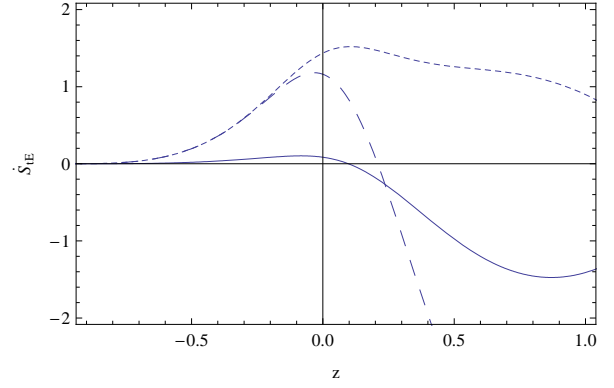


Fig. 1 represents the variation of \dot{S}_{tE} (equation (30)) against redshift z for $w = -2/3$, $\omega = -10$ and $k = 0, \pm 1$. The dashed line, dotted line and filled line represent for $k = 0, -1$ and $+1$ respectively.

IV. STUDY OF THERMODYNAMICS ON THE EVENT HORIZON WITHOUT USING FIRST LAW

Here we assume that the first law is valid only on the apparent horizon. So from equation (27), we have rate of change of entropy on apparent horizon,

$$\dot{S}_A = \frac{4\pi R_A^3 H}{T_A} (\rho_{eff} + p_{eff}) \quad (31)$$

which immediately leads to

$$S_A = \int \frac{4\pi R_A^3 H}{T_A} (\rho_{eff} + p_{eff}) dt \quad (32)$$

where radius of apparent horizon is defined in equation (23). Now we consider the entropy and temperature on apparent horizon in Einstein's gravity i.e.,

$$S_A = \pi R_A^2 \quad (33)$$

and

$$T_A = \frac{1}{2\pi R_A} \quad (34)$$

Using (23), (33) and (34), the equation (31) gives us

$$\dot{H} - \frac{k}{a^2} = -4\pi(\rho_{eff} + p_{eff}) \quad (35)$$

Also eliminating $(\rho_{eff} + p_{eff})$ from (15) and (35) and after integrating we obtain

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3} \rho_{eff} \quad (36)$$

The equations (35) and (36) are the Friedmann equations. So if we consider the first law of thermodynamics is valid on the apparent horizon, we recover the

Friedmann equations in Einstein's gravity.

Another way, if we only consider the entropy defined in (33) rather than the entropy formula (31) from the first law on the apparent horizon, then the derivative of the entropy will be

$$\dot{S}_A = 2\pi R_A \dot{R}_A \quad (37)$$

Also the rate of change of internal entropy on the apparent horizon is (from (29))

$$\dot{S}_{IA} = \frac{4\pi R_A^2}{T_A} (\rho_{eff} + p_{eff}) (\dot{R}_A - H R_A) \quad (38)$$

Adding (37) and (38), and using (11), (12), (25) and (34) and after manipulation we get the rate of change of total entropy on the apparent horizon:

$$\dot{S}_A + \dot{S}_{IA} = \frac{4\pi R_A^2 \dot{R}_A}{T_A} (\rho_{eff} + p_{eff}) \quad (39)$$

which is same as equation (30) on apparent horizon. So we see that the rate of change of total entropy on the apparent horizon by considering first law of thermodynamics is identical with the rate of change of total entropy by considering the entropy formula on the apparent horizon. So we may conclude that the entropy formulae (32) and (33) are equivalent. But on the event horizon, we do not know that this result may or may not hold. Now using first law, the rate of change of total entropy formula for the event horizon has been given in equation (30). In this section we do not consider the first law of thermodynamics while we only consider the entropy and temperature on the event horizon in Einstein's gravity i.e.,

$$S_E = \pi R_E^2 \quad (40)$$

and

$$T_E = \frac{1}{2\pi R_E} \quad (41)$$

Now using (26), (27), (40) and (41), we obtain the rate of change of total entropy for the event horizon:

$$\begin{aligned} \dot{S}_{tE} = \dot{S}_{IE} + \dot{S}_E &= \frac{4\pi R_E^2 \dot{R}_E}{T_E} (\rho_{eff} + p_{eff}) \\ &+ 2\pi R_E (H R_E - 1 - 4\pi R_E^3 H) \end{aligned} \quad (42)$$

which implies

$$\begin{aligned} \dot{S}_{tE} = \dot{S}_{IE} + \dot{S}_E &= -8\pi^2 R_E^3 (\rho_{eff} + p_{eff}) \\ &+ 2\pi R_E (H R_E - 1) \end{aligned} \quad (43)$$

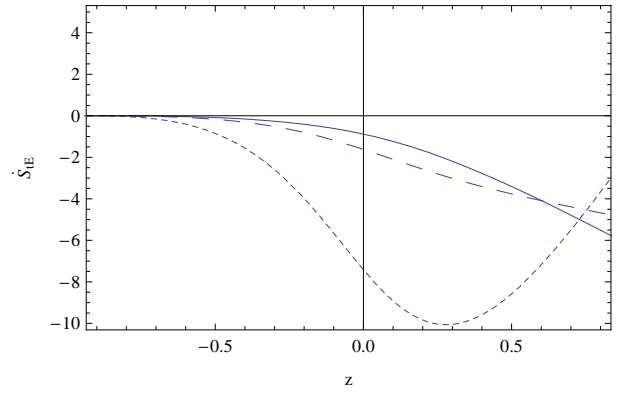


Fig. 2 represents the variation of \dot{S}_{tE} (equation (43)) against redshift z for $w = -2/3$, $\omega = -10$ and $k = 0, \pm 1$. The dashed line, dotted line and filled line represent for $k = 0, -1$ and $+1$ respectively.

Comparing equations (30) and (42), we see that there is an extra term (positive or negative) arises in (42) than (30). So the rate of change of total entropy on the event horizon by considering first law of thermodynamics is not identical with the rate of change of total entropy by considering the entropy formula on the event horizon. So we may conclude that the entropy formulae (27) and (40) are not equivalent on the event horizon. Or shortly speaking, if first law is valid on event horizon then entropy formula (40) is not valid on the event horizon or, if entropy formula (40) is valid on the event horizon then first law cannot be satisfied on the event horizon. If r.h.s of (43) is non-negative then GSL is satisfied on the event horizon, which depends on the BD scalar field solutions. From (13) and (14), we see that $(\rho_{eff} + p_{eff})$ is independent of V . So the validity of GSL depends only on matter and scalar field, but not its potential. Now using BD solutions (17)-(20), we draw the rate of change of total entropy on the event horizon i.e., \dot{S}_{tE} against z in figure 2 for open, closed and flat models. From the figure it is to be seen that when z decreases, \dot{S}_{tE} becomes negative. So we may conclude that GSL of thermodynamics is not satisfied for the BD solutions.

V. DISCUSSIONS

In this work, we have described the modified version of Brans-Dicke theory of gravity and given a particular solution by choosing a power law form of scalar field ϕ and constant ω . We have examined the validity of GSL on apparent and event horizons in BD theory by an effective approach. Next, assuming first law of thermodynamics, the validity conditions of GSL on event horizon are presented. If we assume first law and entropy formula on the apparent horizon then using continuity equation of matter fluid, we can recover two Friedmann equations. Also without use first law if we impose the entropy relation on the horizon, then we also obtain the condition of validity of GSL on the event horizon. There are two ways to get validity conditions of GSL on apparent and event horizons: (i) use first law

and find entropy relation on the horizons (ii) use horizon entropy on the horizons. On the apparent horizon the two process are equivalent, but on the event horizon they are not equivalent. We have also shown that the GSL on the apparent horizon is always valid in BD theory also. We have also shown that the GSL do not depend on the scalar field potential in BD theory. From fig.1 and fig.2, we see that there are opposite behaviours of \dot{S}_{tE} , i.e. if first law is valid on event horizon then GSL is successfully satisfied for BD solution, but if first law is not valid then the GSL is no longer satisfied for BD solution. So we may conclude that first law always favours GSL of thermodynamics in BD theory. In our effective approach, the first law and GSL is always satisfied in apparent horizon, which do not depend on BD theory of gravity.

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